# Relativity and Space Geodesy

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***IAU General Assembly, Prague, 21st August 2006***

# Outline of the speach

## I. Native relativistic approach *wrt* spacecraft trajectory :

orbitography

**a. Needed in:** precise planetary gravitational field modeling, orbitography

**b. Illustration:**

classical *vs* RMI prototype –Relativistic Motion Integrator-method

[ Pireaux, Barriot, Rosenblatt, Acta A 2005]

## II. Native relativistic approach *wrt* photon trajectory:

[ Pireaux et Barriot, Cel. Meca en prépa]

laser-links (time transfer, frequency shift)

**a. Needed in:** LISA, Tippo, T2L2, Galileo …

**b. General method for relativistic laser-links**

**c. Illustration:** LISA

[B. Chauvineau, T. Régimbau, J.-Y. Vinet, S. Pireaux, Phys. Rev. D 72, 122003 (2

## III. Caution with relativistic time-scales

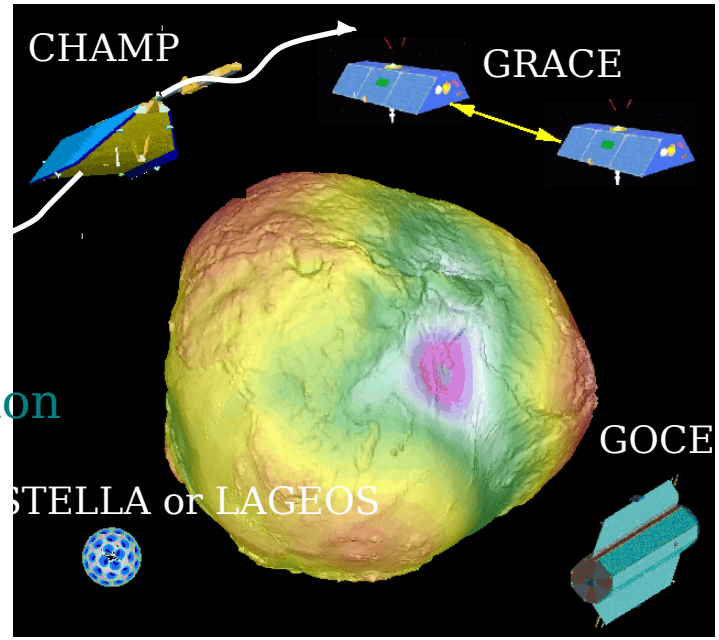
**a. Relativistic time-scales**

**b. Illustration:** LISA

# I. Native relativistic approach *wrt* spacecraft trajectory : orbitography

## Ia. Needed in: - precise planetary gravitational field modeling

- orbitography




- **A good planetary gravitational field model?**
  - ➡ precise orbitography
  - ➡ good model of perturbations
  - ➡ relativistic gravitation:
    - Schwarzschild precession
    - geodesic ‘’
    - Lense-Thirring ‘’

- **Include IAU 2000 standards regarding General Relativity:**
  - GCRS metric
  - time transformation
  - Earth rotation
  - ...

# Ib. Illustration:

## classical method:

numericaly integrate **Newton's second law** of motion:

$$\vec{A} = \overrightarrow{\mathrm{grad}}U_E + \vec{A}_{\text{Perturbating Bodies + E}} + \vec{A}_{\text{Atmospheric Drag}} + \vec{A}_{\text{Radiation Pressure}} + \overrightarrow{\mathrm{grad}}U_{\text{Earth Tides}} + \overrightarrow{\mathrm{grad}}U_{\text{Ocean Tides}} + \vec{A}_{\text{Relativist}} + \vec{A}_{\text{Atmospheric Pressur}}$$

## RMI (Relativistic Motion Integrator) prototype method:

numericaly integrate **relativistic** equation of motion (for a given metric):

$$\frac{dV^\alpha}{d\tau} = -\Gamma^\alpha_{\beta\gamma} V^\beta V^\gamma + K_\beta \left( G^{\alpha\beta} - V^\alpha V^\beta \right)$$

with

- $K_\alpha \equiv$ quadri-"force"
- $X^\alpha = (T, X, Y, Z)$
- $\alpha, \beta, \gamma = 0, 1, 2, 3$
- $\tau$ = proper time
- $\Gamma^\alpha_{\beta\gamma}$ = Christoffel symbol *wrt* GCRS metric $G_{\alpha\beta}$

$$V^\alpha \equiv \frac{dX^\alpha}{d\tau}$$ and first integral $G_{\alpha\beta} V^\alpha V^\beta = c^2$

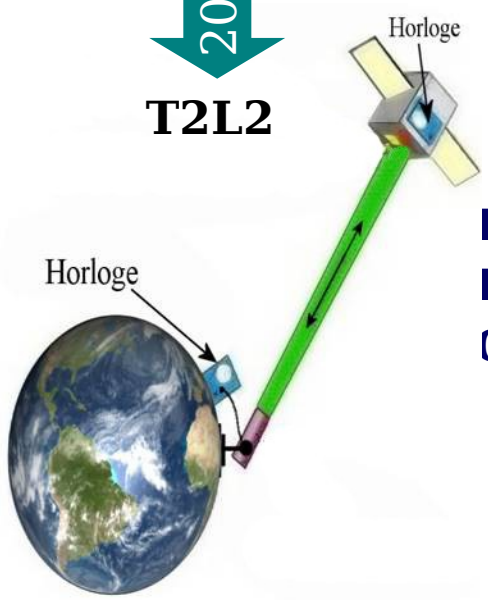
Simplectic

# II. Native relativistic approach *wrt* photon trajectory: laser-links

## . Need for relativistic laser links:

2008

**T2L2**




**Project:** CNES

**Implied:** GEMINI/OCA

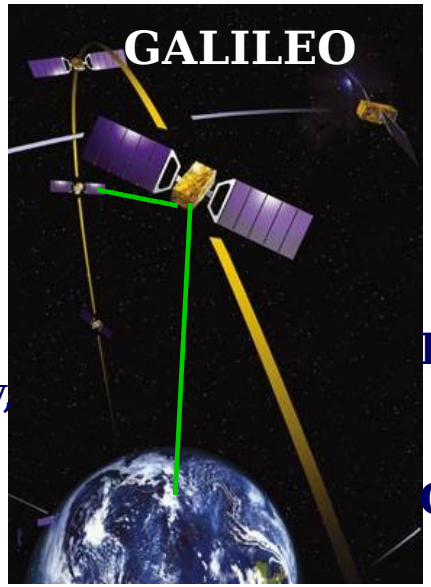
**Goals:** metrology, geodesy, clocks synchro. …

2008-2012

**Project:** CNES, ESA, CE

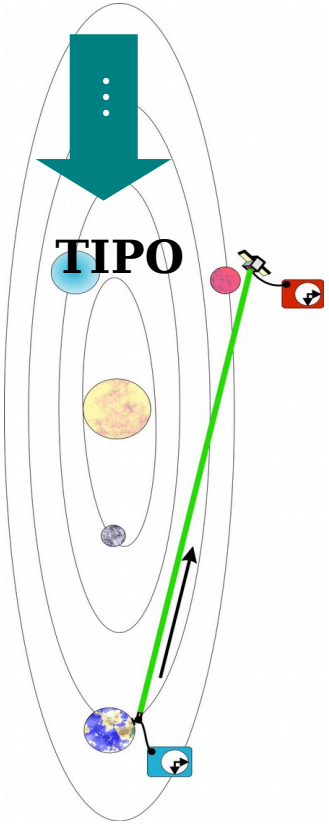
**Implied:** GEMINI/ OCA

**Goals:** positioning, …




…

**TIPO**



**Implied:** GEMINI, ARTEMIS, through SIR ILIADE of OCA

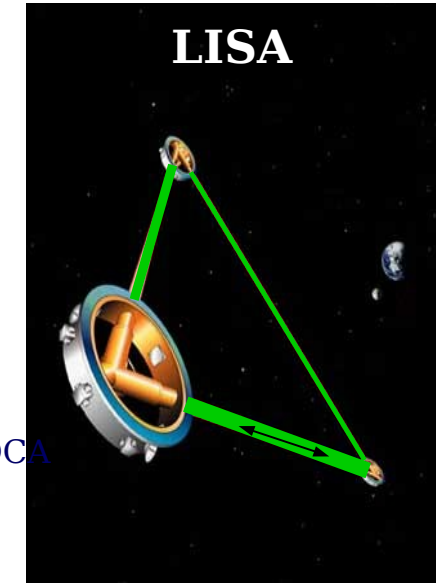
**Goal:** metrology, planetodesy, …

2014-2020

**Project:** CNES, ESA, NASA

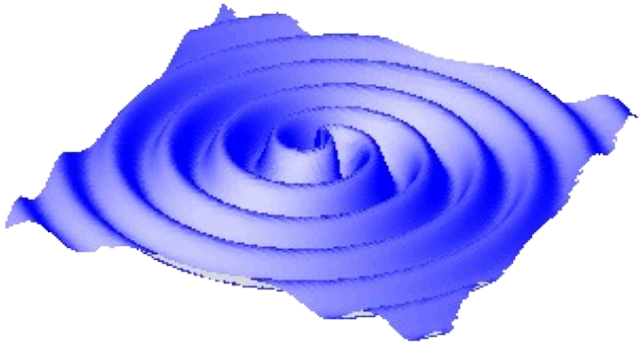
**Implied:** LISAFrance

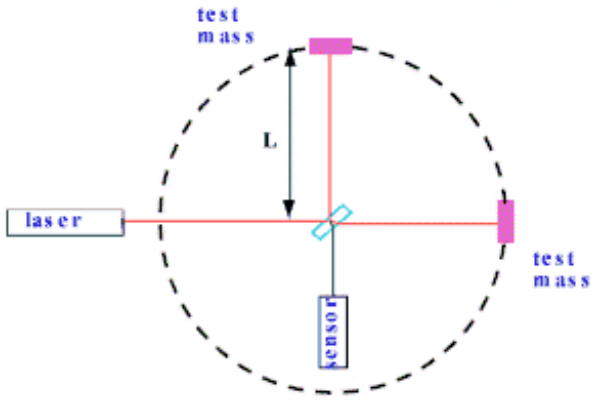
**Goals:** Time Delay Interferc



# LISA (Laser Interferometer Space Antenna)

- **LISA = space GW detector complementary to ground detectors**





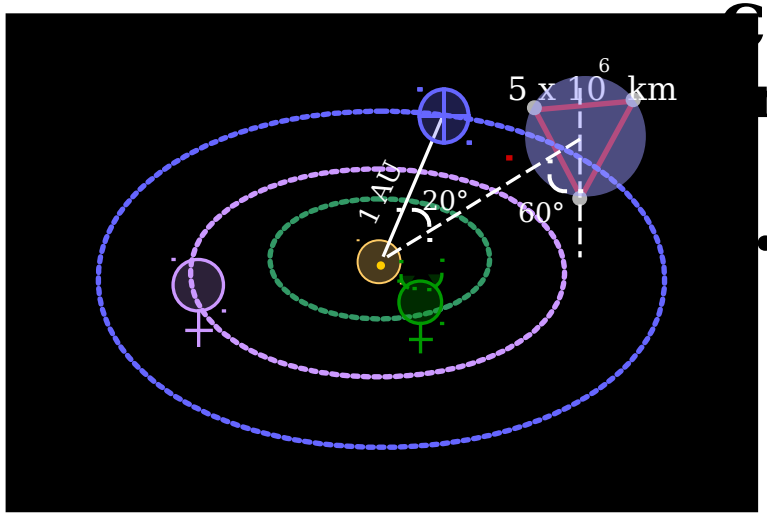

- **GW detection through measurement of phase shift due to $\Delta L$**
  - ➡ good precision required on arm length: $\Delta L/L \sim 10^{-23}$

**➡ laser frequency noise and optical bench noise >>> GW signal**

➡ TDI pre-processing of data required

➡ TDI observables = time-delayed (*wrt* photon flight time $t_{ij}$) combinations of data fluxes from 6 laser links, in close loops, in order to cancel bench and frequency noise

# ativistic modeling of orbitography/laser links required:



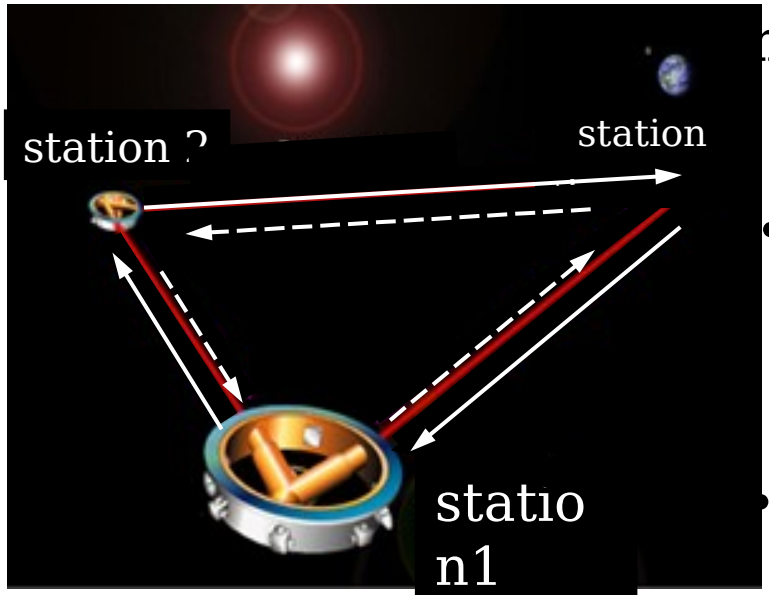

**Coordinates of stations ?**
**nterdistance ($L_{ij}$) ?**

- △ equilateral
- 5 million km interdistance

$L_{ij}(t)$

- rotation of △
- rotation around ☉
- at 20° behind ⊕
- planets present

gravitational relativistic effects




**hoton travel time ($t_{ij}$) ?**

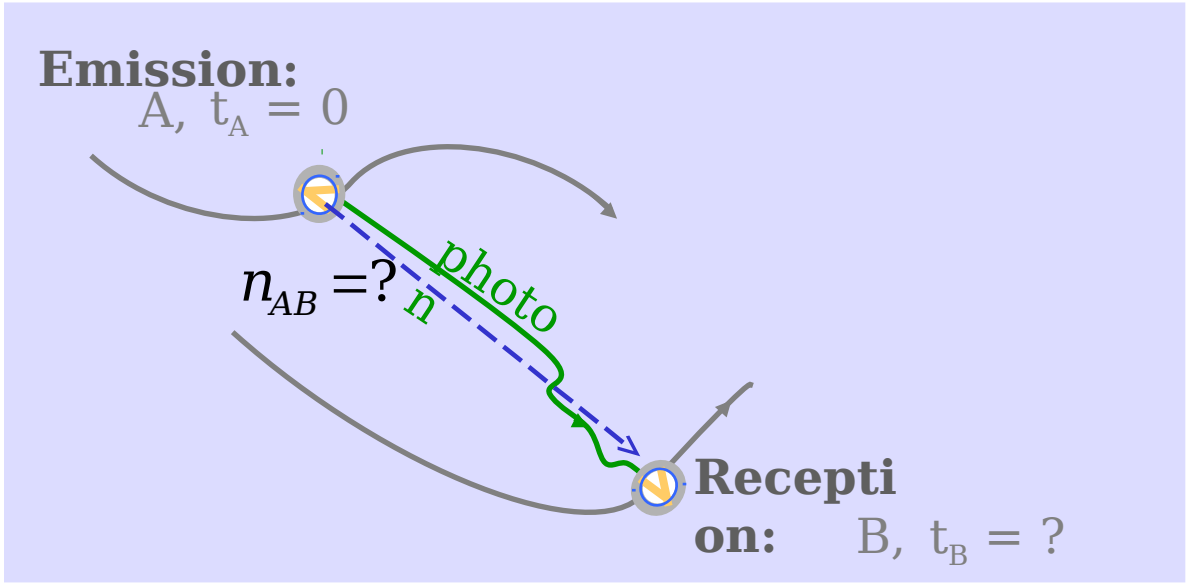
- double laser links
- 3 (drag-free) stations
  3 test masses

geodesic motion ⇓ classical doppler, Sagnac effect…

- planets and ☉ present → light deflection…

# IIb. General method for relativistic laser-links

- **Laser link:**




## Equation to be solved in terms of quantities at $t_A$

$$x_{ph}(t_B, n_{AB}, x_{A0}) = x_B(t_B, x_{B0}, v_{B0})$$

Photon orbit — Receiving station orbit

➡ $(t_{AB} = t_B - t_A, n_{AB})$ (flight time, « direction ») = 1 + 2 (normalization) = 3 unkno

# Motion in background metric $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$ presence of gravitational sources (sce) :

$ds^2 = g_{\alpha\beta}\, dx^\alpha\, dx^\beta$ … with IAU2000 conventions

$$= \left(-1 + \sum_{sce\,i} \frac{2GM_i}{r_i\, c^2} + O\left(\frac{1}{c^4}\right)\right) c^2 dt^2$$

$$- \left(\sum_{sce\,i} \frac{4GM_i V_i^k}{r_i\, c^3} + O\left(\frac{1}{c^5}\right)\right) c\, dt\, dx^k$$

$$+ \left(1 + \gamma \sum_{sce\,i} \frac{2GM_i}{r_i\, c^2} + O\left(\frac{1}{c^4}\right)\right) \left(dx^2 + dy^2 + dz^2\right)$$

- **Proper- *vs* coordinate-time rates:** $dt = \sqrt{-ds^2}\,/c$

$$= dt\left(1 + O\left(\frac{1}{c^2}\right) + O\left(\frac{1}{c^3}\right) + O\left(\frac{1}{c^4}\right) + \ldots\right)$$

- **Proper *vs* coordinate time:** $t = \tau + \ldots$

**Energy measured from spacecraft** $E = -\, g_{\alpha\beta}\, k^{\alpha}\, u^{\beta}$

where $u^{\alpha} = dx^{\alpha}/d\tau$ = spacecraft 4-velocity

$v^{i} = dx^{i}/dt = u^{i}/u^{0}$

$k^{\alpha} = dx^{\alpha}/d\lambda$ photon 4-wave vector

**equency shift =** $z_{AB}(t_A) = \dfrac{E_B(t_B = t_A + t_{AB})}{E_A(t_A)} - 1$

= relative difference between (if transfer from A to B)

frequency of photon, emitted by A, measured when receiv

proper frequency of photon when emitted by A
(= proper frequency of identical oscillators aboard A and

# tributions from gravitational sources (sce) to $h_{\alpha\beta}$ :

**rder 1** :

terms in $\sim \frac{2GM}{rc^2}$

→ Central body : presence, shape, orbital motion (during photon travel t

→ Other bodies : presence, orbital motion

orbital motion: $\delta\left[\frac{2GM}{rc^2}\right] \sim \frac{2GM}{rc^2}\frac{V_{sce}\delta t_{vol}}{r}$

➢ **Order 3/2** :

terms in $\sim 2\frac{2GM}{rc^2}\frac{V_{sce}}{c}\frac{dx}{c.dt}$

→ Central body: rotation, orbital motion

→ Other bodies: orbital motion

with $\frac{dx}{c.dt}$ = 1 for photons, $\rightarrow V_{sat}/c$ for satellites

➢ **Order 2** : terms in $\sim \left[\frac{2GM}{rc^2}\right]^2$

# IIc. Illustration: LISA, rotation around the Sun

## Orders of magnitude :

$$h_{\alpha\beta} = h^{(1)}_{\alpha\beta} + h^{(3/2)}_{\alpha\beta} + h^{(2)}_{\alpha\beta} + \dots$$

$\sim 10^{-16}$

**Sun rotation:** (<<) $\sim 10^{-13}$

**Orbital motion of sces:**

Sun $\sim 10^{-15}$

Jupiter $\sim 2 \cdot 10^{-16}$

Venus $\sim 10^{-17}$

$$= h^{(1)(\text{Sun})}_{\alpha\beta} + h^{(1)(\text{othebodies})}_{\alpha\beta}$$

**Presence:** $\sim 2 \cdot 10^{-12}$

**Orbital motion:** $\sim 10^{-18}$

**Presence:** $\sim 10^{-8}$

**Orbital motion:** $\sim 2 \cdot 10^{-16}$

$\sim 50$ m

$\sim 2 \cdot 10^{-7}$ s

**Photon flight:**
**$5 \cdot 10^{+6}$ km**

# LISA Flight time solution:

$$t_{AB} = {}^{0}t_{AB} + {}^{1/2}t_{AB} + {}^{1}t_{AB} + O(\varepsilon^{3/2})$$

**r 0 :** $${}^{0}t_{AB} = \frac{L_{AB0}}{c}$$ where $$L_{AB0} = \pm \left| x_{B0} - x_{A0} \right|$$ (+ sign : photon travels fro

Classical

evaluated at $t_A$

**order 1/2 :** $${}^{1/2}t_{AB} = {}^{0}t\,\frac{\vec{n}_{AB}\cdot\vec{v}_{B0}}{c}$$ where $$\vec{n}_{AB} = \frac{x_{B0} - x_{A0}}{L_{AB0}}$$

Classical kinematic terms

➢ **order 1:** $${}^{1}t_{AB} = \frac{1}{2}{}^{0}t\left[\frac{v_{B0}^2}{c^2} + \left(\frac{\vec{n}_{AB}\cdot\vec{v}_{B0}}{c}\right)^2\right] + \vec{n}_{AB}\left[(1+\gamma)\frac{GM}{c^3}\vec{\chi}\left({}^{0}t, \vec{n}_{AB}\right)\right] - \frac{GM\,\vec{x}_{B0}}{2r_{B0}^3 c}\,{}^{0}t^2$$

Kinematic terms

Shapiro delay

Velocity change during photon flight time

$$\vec{\chi}(t, \vec{n}_{AB}) = (r - r_0)\vec{P} + \vec{n}_{AB}\ln\frac{n_{AB}\cdot x_{A0} + ct + r}{n_{AB}\cdot x_{A0} + r_0}$$

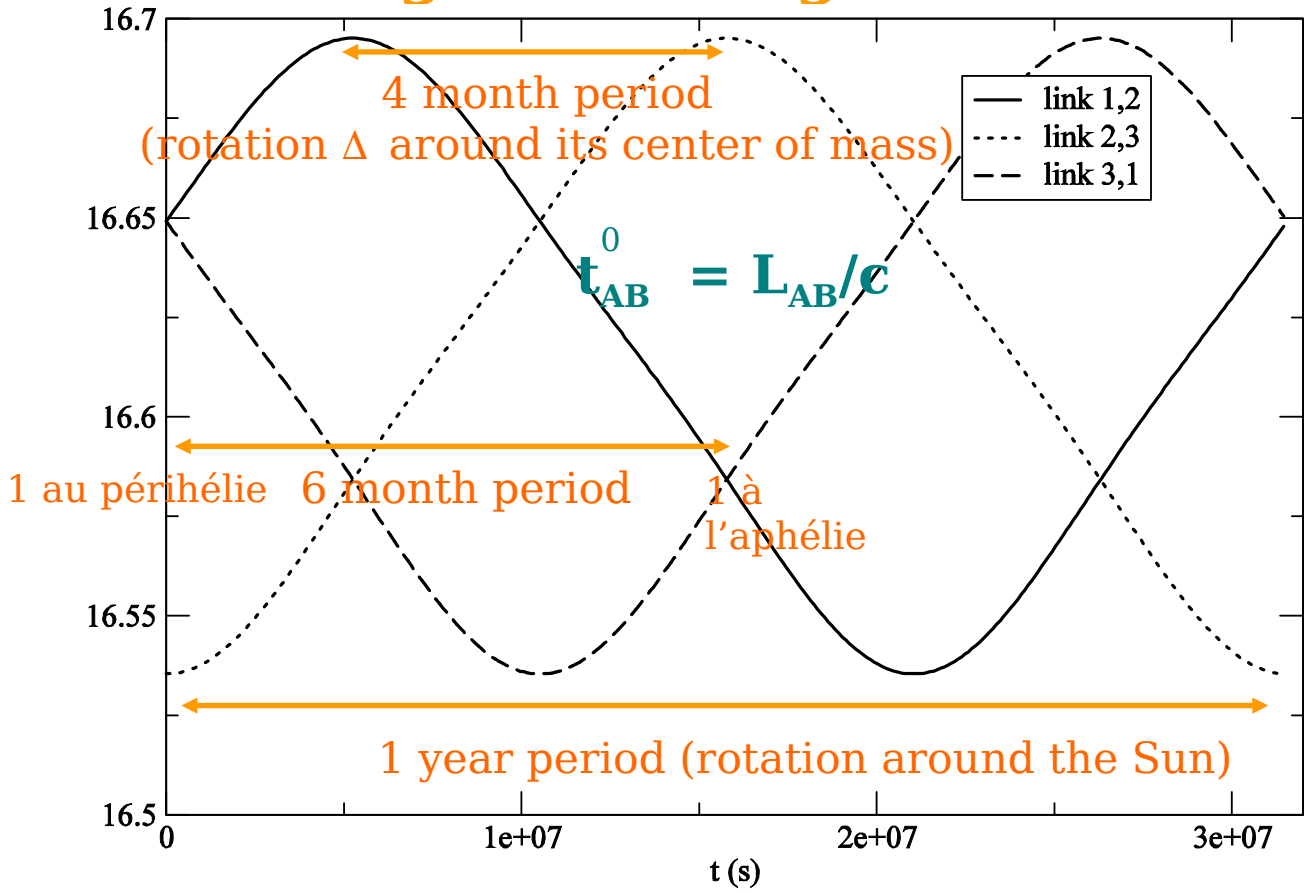
where

$$\vec{P} = \frac{x_{A0} - (n_{AB}\cdot x_{A0})\,n_{AB}}{r_{A0}^2 + (n_{AB}\cdot x_{A0})^2}$$

# rical estimates of geometric time delays in s over

**er 0 :** amplitude ~ 48 000 km/c

## « flexing » of triangle

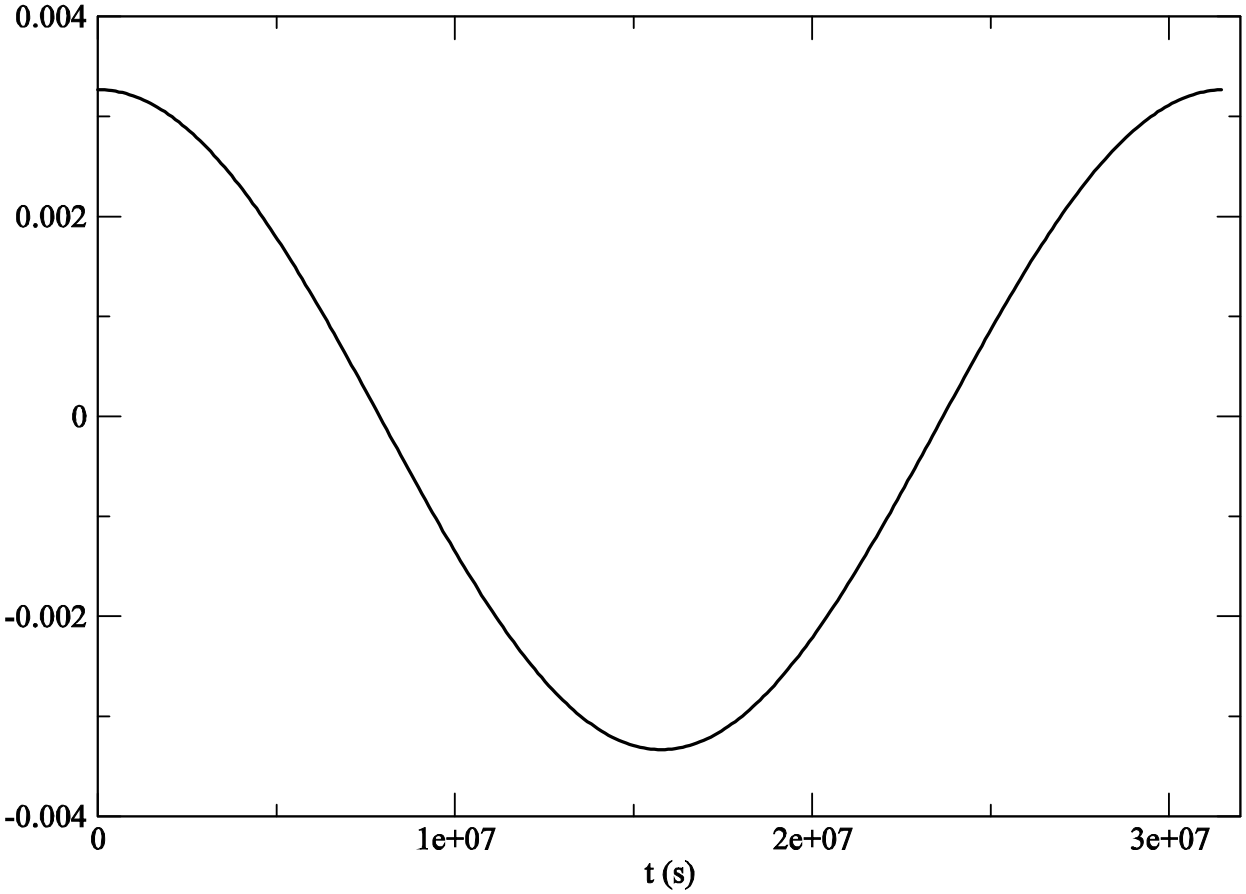


# rical estimates of geometric time delays in s over a

**er 0 :** « flexing » of triangle, amplitude ~ 48 000 km/c ;

**er 1/2 :** amplitude ~ 960 km/c ;

$t_{23}$-$t_{32}$ is not symmetric $t_{AB}$ **(Sagnac+aberration term)**



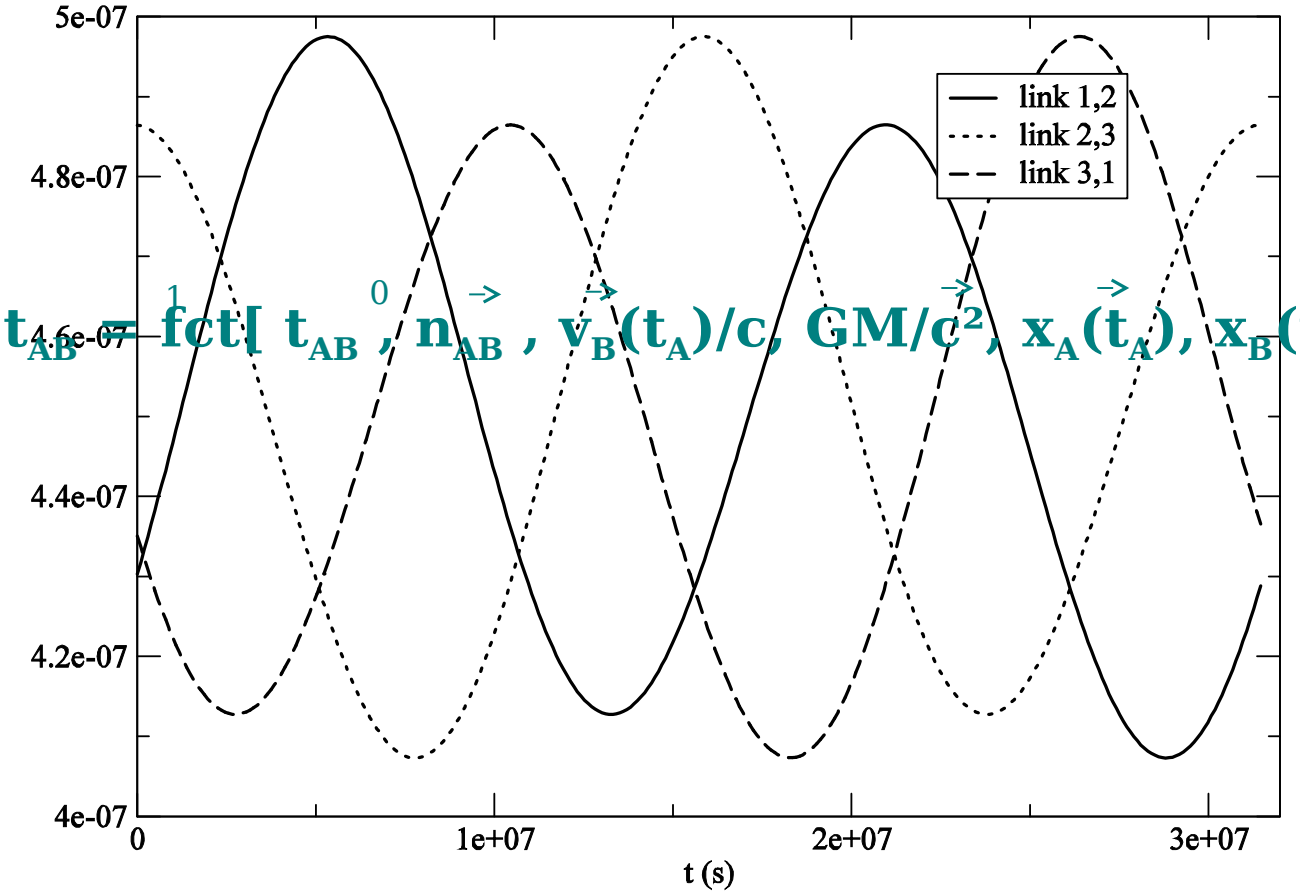
# rical estimates of geometric time delays in s over a

**er 0 :** « flexing » of triangle, amplitude ~ 48 000 km/c ;

**er 1/2 :** spacecraft Doppler, amplitude ~ 960 km/c ;

**er 1 :** less than 30 m/c.

**relativistic gravitational Einstein, Doppler, Shapiro effects**




$$t_{AB} = fct[\ t_{AB}^{1},\ t_{AB}^{0},\ \vec{n}_{AB},\ \vec{v}_B(t_A)/c,\ GM/c^2,\ \vec{x}_A(t_A),\ \vec{x}_B(t_A)\ ]$$

# • LISA Frequency shift solution:

$$z_{AB} = {}^{1/2}z_{AB} + {}^{1}z_{AB} + {}^{3/2}z_{AB} + O(\varepsilon^2)$$

$\approx 2\,10^{6}$ ?

$\approx 2\,10^{10}$ ?

$\approx 2\,10^{14}$?

➢ **Naive estimate:**

$${}^{n}z_{AB} \sim \varepsilon^{n}\,\frac{L}{r}\cos\frac{\pi}{3} \sim \frac{(10^{-8})^{n}}{60}$$

➢ **Order 1/2:** ${}^{1/2}z = -\vec{n}.\frac{v_{B0} - v_{A0}}{c}$

Kinematic terms (Doppler)

$\sim 7\,10^{-8}$

LISA configuration

(spacecraft orbits: circular about CM

+

velocity proportional to orbital radius)

=> (reduction factor ~ L/R)

- **Order 1:**

$$z^{1} = \vec{n}.\frac{\vec{v}_{B0} - \vec{v}_{A0}}{c} - \frac{1}{2}\left(\frac{\vec{v}_{B0} - \vec{v}_{A0}}{c}\right)^2 + \frac{GMt}{c}\frac{\vec{n}^{0}_{AB}.\vec{x}_{B0}}{r^3_{B0}} + \frac{GM}{c^2}\left(\frac{1}{r_{B0}} - \frac{1}{r_{A0}}\right)$$

Kinematic terms

Velocity change during photon flight time

Einstein effect

$\sim 2\,10^{15}$

$\sim 6\,10^{12}$

$\sim 2\,10^{10}$

$\sim 2\,10^{10}$

$\sim 6\,10^{12}$

L<<R=> compensation (reduction factor ~

$$= \frac{GM}{c^2}\left(1 - 3\left(\frac{\vec{n}^{0}_{AB}.\vec{x}_{B}}{r_B}\right)^2\right)\frac{L^2}{2r^3_B} + O\left(\frac{L^3}{r^4}\right)$$

$\sim 2\,10^{13}$

free fall + LISA configuration (~ 60°) => compensatio

$$= -\frac{1}{2c^2}\frac{d}{dt}\left(L\,\vec{n}^{0}_{AB}.(\vec{v}_B - \vec{v}_A)\right) + O\left(\frac{L^3}{r^4}\right)$$

# • LISACODE

collaboration of **ARTEMIS (Côte d'Azur) – APC (Paris),** in LISA FRANCE

aims at:
- mission simulations
- Tests of TDI data pre-processing, TDI-ranging
- sensitivity curves
- relevant order of magnitude estimates
- ...

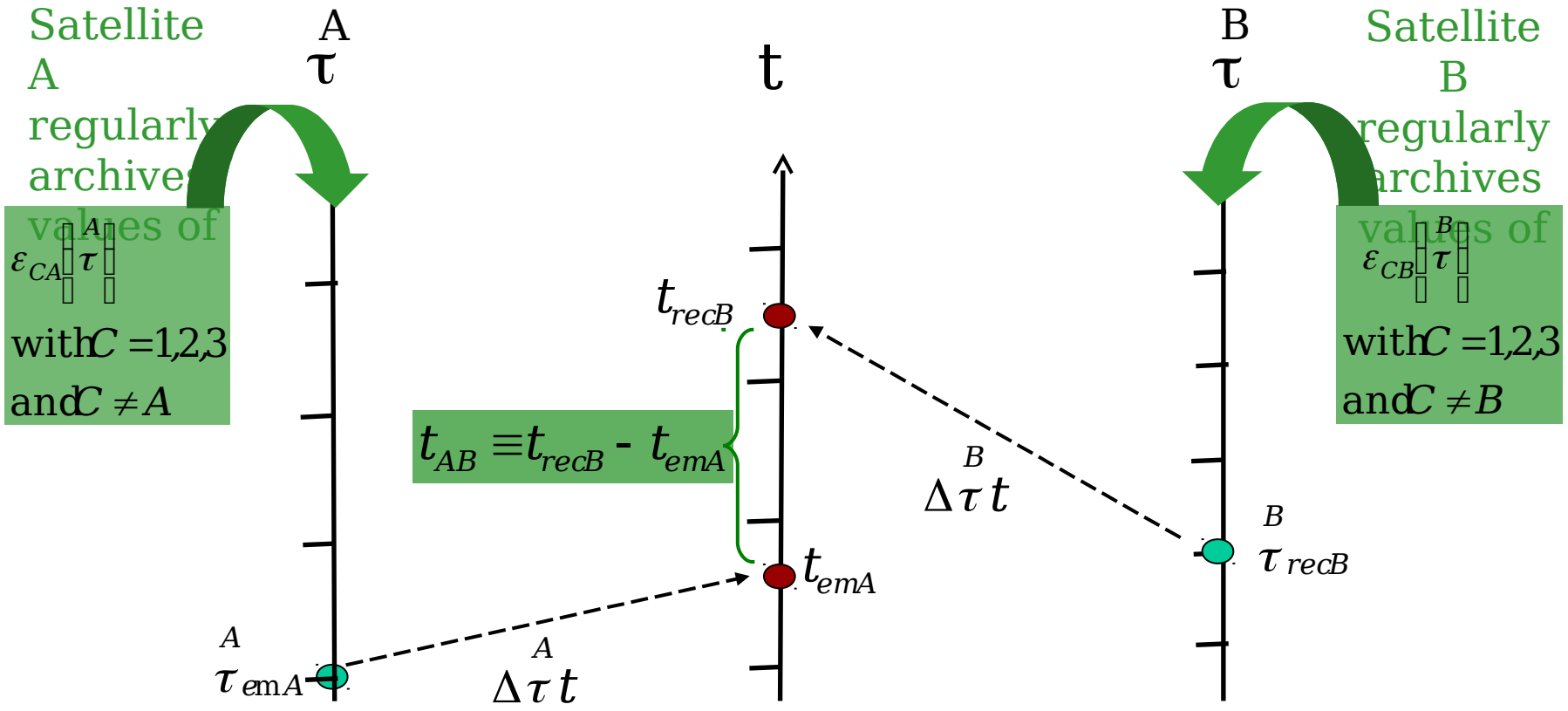
includes:
- *relativistic* laser links (time transfer + freq. shift)
- *classical* orbito.

without planets
- *coordinate time only*

- **Laser link :** Sun alone sufficient,
  but relativistic description of its field necessary
- **Ephemeris of stations :** presence of planets necessary,
  to provide initial conditions for photon flight times
- **Time scales:** careful with archives and coherence

# III. Caution with relativistic time-scales

## IIIa. Time scales




Proper time of satellite A **(physical scale)**

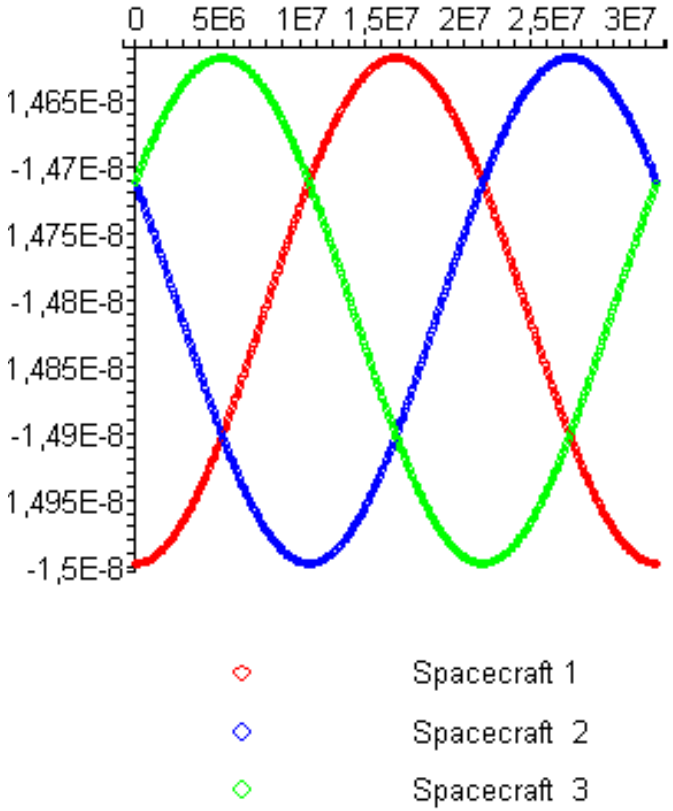
Barycentric coordinate time **(artificial scale)**

Proper time of satellite B **(physical scale)**

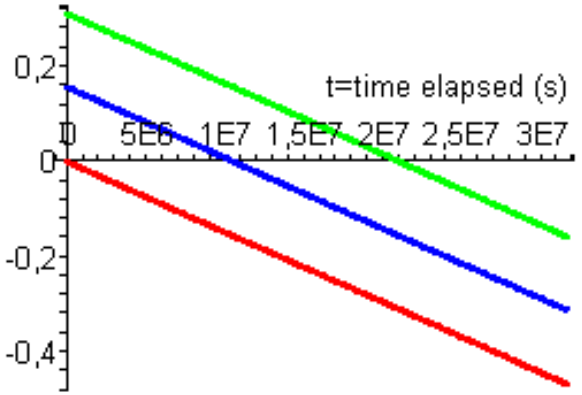
# IIIb. Illustration: LISA

## Numerical estimates over a one year mission...

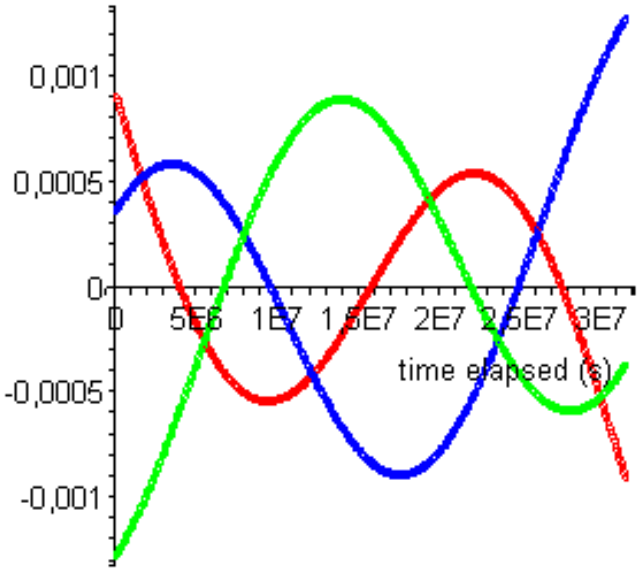
$d\tau^A/dt - 1$




$\tau^A - t$ (s)




$\tau - t^A$ (s) linear trend removed



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## III. Caution with relativistic time-scales

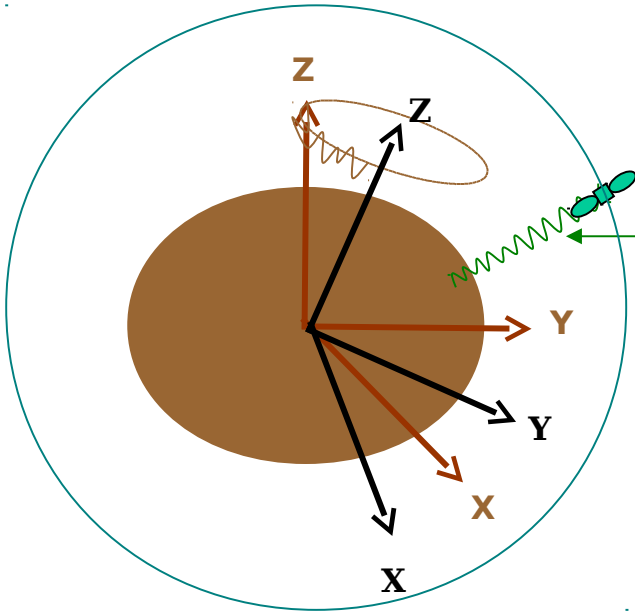
**a. Relativistic time-scales**

**b. Illustration:** LISA

# Other transparencies

# **Geodesy:** precise geophysics implies precise geodesy

Satellite motion current description: Newton's law + relativistic corre + other forces




Relativistic corrections on measurements

(X,Y,Z) = planetary crust frame ← *Planetary potential mode*

*Planetary rotation model*

(X,Y,Z) = quasi inertial frame ← *Satellite motion*

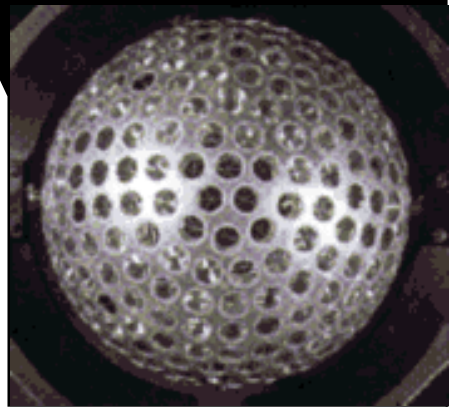
rors in relativistic corrections, time or space transformations…

s-modeling in the planetary potential or the planetary rotation model

➡ **better use relativistic formalism directly**

# Geodesy examples:
## a high-, or respectively low-altitude satellite...

LAGEOS 1



### Laser GEOdymics Satellite 1

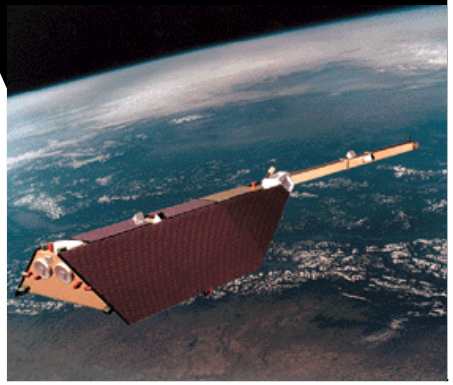
**Aims:** - calculate station positions (1-3cm)
- monitor tectonic-plate motion
- measure Earth gravitational field
- measure Earth rotation

**Design:** - spherical with laser reflectors
- no onboard sensors/electronic
- no attitude control

**Orbit:** **5858x5958km**, i = 52.6°, around Earth

**Mission:** 1976, ~50 years (USA)

CHAMP



### CHAllenging Minisatellite Payload

**Aims:** - precise gravity and magnetic field, their space and time variations

**Design:** - laser reflector, GPS receiver
- drift meter
- magnetometer, star sensor, accelerome

**Orbit:** **454 km initial**, near polar, around Earth

**Mission:** ~5 years (Germany)

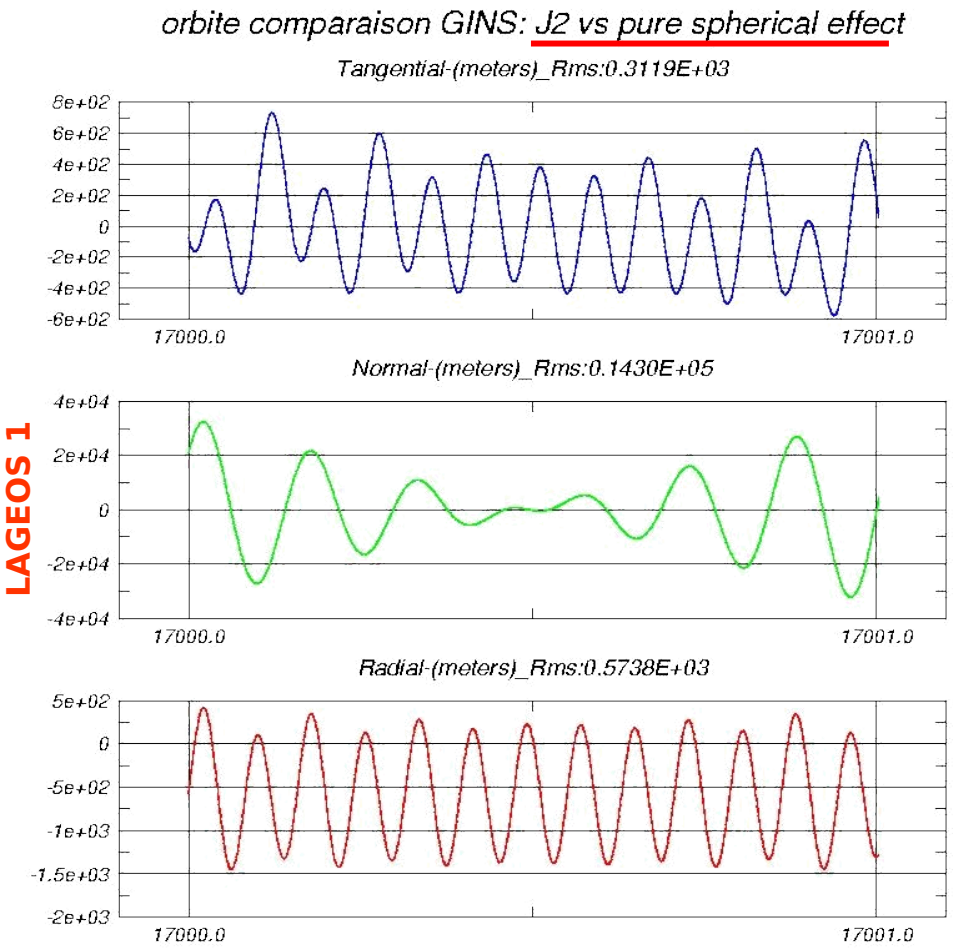
# Geodesy: orders of magnitude [m/s²]

| Cause | LAGEOS 1 | CHAMP |
|---|---|---|
| Earth monopole | 2.8 | 8.6 |
| Earth oblateness | 1.0 10**-3 | 1.1 10 **-2 |
| Low order geopotential harmonics (eg. l=2,m=2) | 6.0 10**-6 | 6.4 10**-5 |
| High order geopotential harmonics (eg.l=18,m=18) | 6.9 10**-12 | 9.4 10**-7 |
| Moon | 2.1 10**-6 | 7.9 10**-7 |
| Sun | 9.6 10**-7 | 2.7 10**-7 |
| Other planets (eg. Ve) | 1.3 10**-10 | 9.8 10**-13 |
| Indirect oblation (Moon-Earth) | 1.4 10**-11 | 1.4 10**-11 |
| **General relativistic corrections (total)** | **9.5 10**-10** | **1.7 10**-8** |
| Atmospheric drag | 3 10**-12 | 3.5 10**-7 |
| Solar radiation pressure | 3.2 10**-9 | 3.2 10**-8 |
| Earth albedo pressure | 3.4 10**-10 | 3.3 10**-9 |
| Thermal emission | 1.9 10**-12 | 8.3 10**-9 |

**High satellite** **Low satellite**

# Gravitational potential model for the Earth

$$U_E = \frac{\overline{GM}_E}{|X|} \sum_{l=0}^{l_{max}} \sum_{m=0}^{l} \left[ \frac{\Re_E}{|X|} \right]^l \bar{P}_{lm}(\sin\varphi)\left[\bar{C}_{lm}\cos m\lambda + \bar{S}_{lm}\sin m\lambda\right]$$



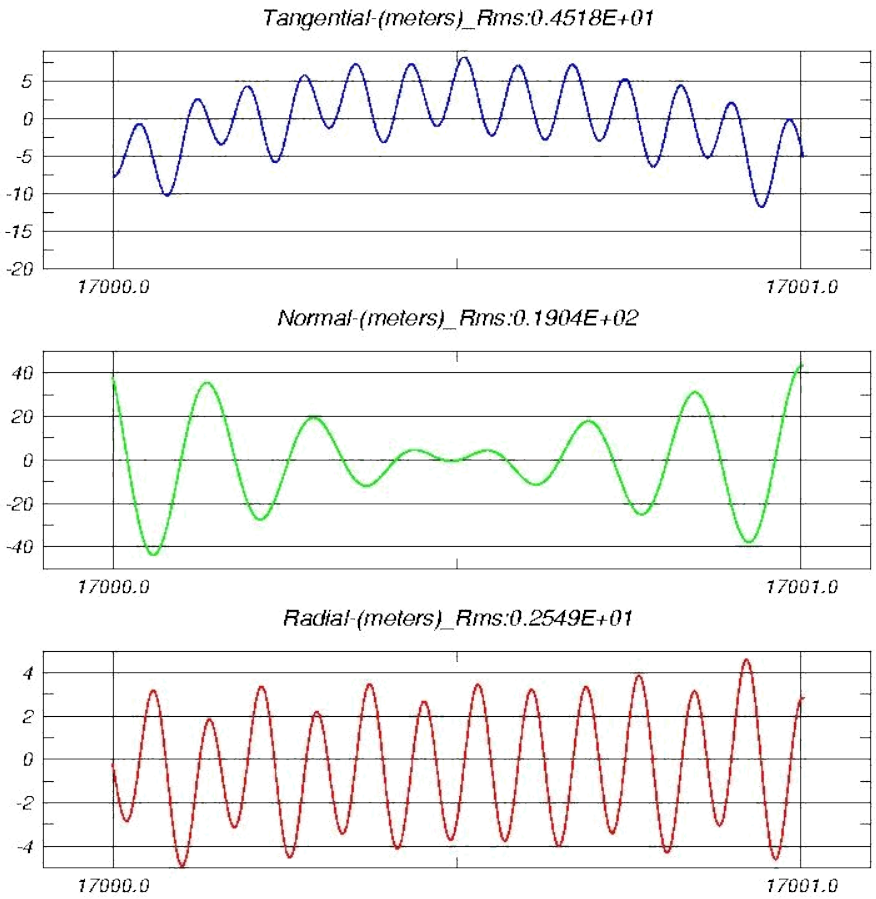
# ewtonian contributions from the Moon, Sun and Plan

$$\vec{A}_{PB\neq E} = \vec{A}_{J2\text{-}Mooncoupling} + \sum_{body\,"n"} \vec{A}_{3rdbody_n}$$

with

$$\vec{A}_{J2\text{-}Mooncoupling} = -\frac{3}{2}\frac{GM_{Mo}}{\left|X_{Mo}\right|^5}\sqrt{5}\,\overline{C}_{20}\,\Re_E^2 \begin{bmatrix} 5\frac{X_{Mo}^2}{\left|X_{Mo}\right|^2} - 1 \\ 0 \\ 0 \end{bmatrix} \vec{X}_{Mo} - 2X_{Mo} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

*orbite comparaison GINS: Moon perturbation (3rd body + J2 coupling)*




LAGEOS 1

and

$$\vec{A}_{3rdbody_n} = -GM_n \left[ \frac{\vec{X} - \vec{X}_n}{\left|\vec{X} - \vec{X}_n\right|^3} + \frac{\vec{X}_n}{\left|\vec{X}_n\right|^3} \right]$$

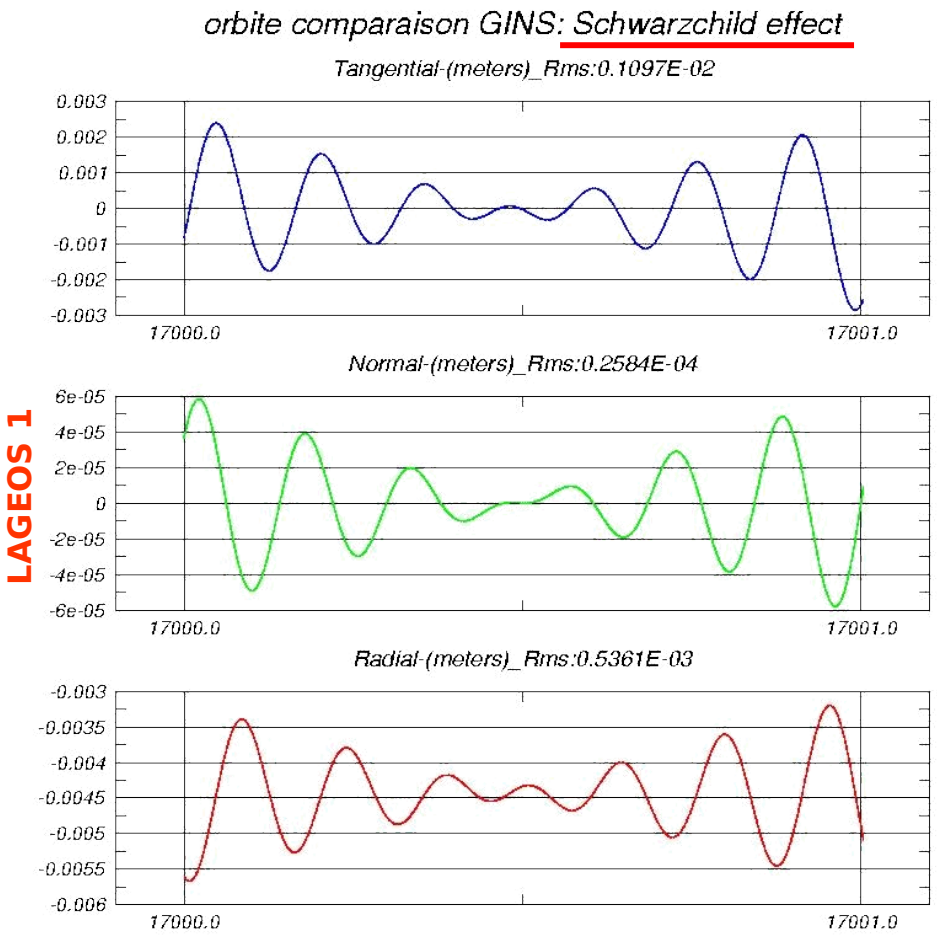
$$\begin{bmatrix} +0.58286072 \\ -1.02761036 \\ +0.34965593 \end{bmatrix} \cdot 10^6 \ m/s^2$$

$XYZ_0$

# c) Relativistic corrections

$$\vec{A}_R = \vec{A}_{Schwarzschild} + \vec{A}_{Geodetic(DeSitter)\ Precession} + \vec{A}_{Lens\text{-}Thirring\ Precession}$$

$$\vec{A}_{Schw} = \frac{GM_E}{c^2|X|^3}\left[\frac{4GM_E}{|X|} - |\vec{V}|^2\right]\vec{X} + 4(\vec{V}\cdot\vec{X})\vec{V}$$

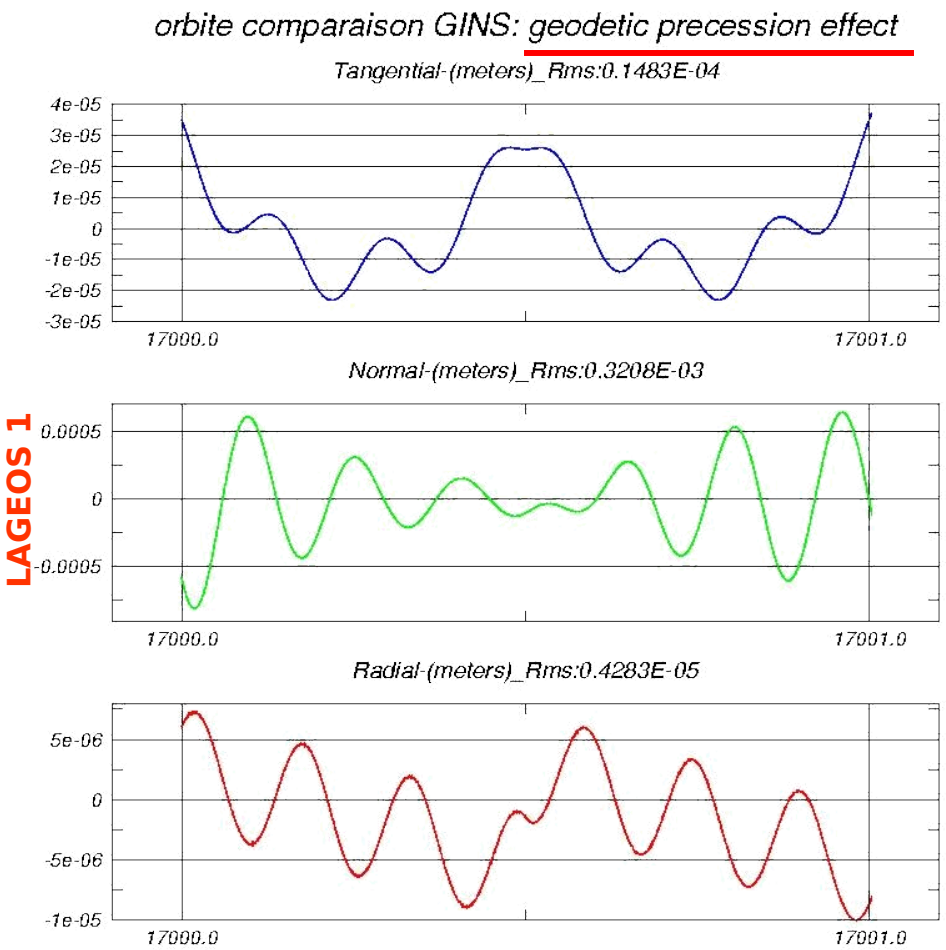



LAGEOS 1

$$\begin{bmatrix} +0.187604 \\ +4.524321 \\ -0.210319 \end{bmatrix} \cdot 10^{8}\ m/s^2$$

$XYZ_0$

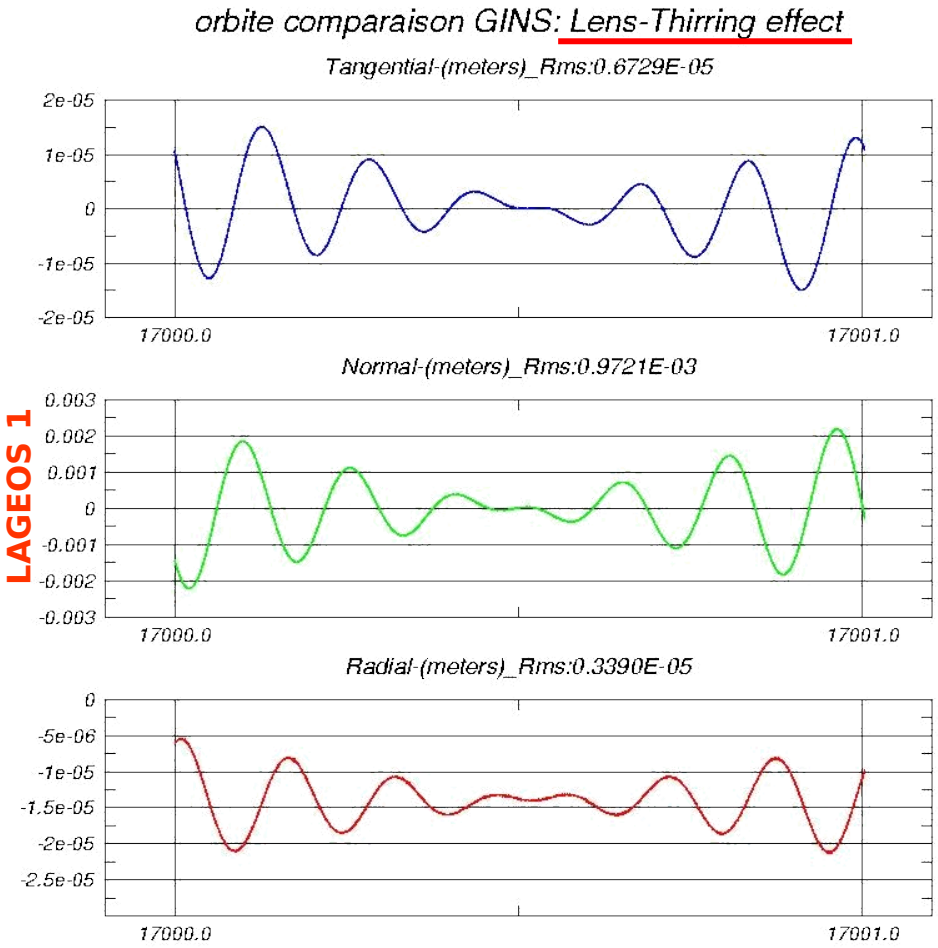
$$\vec{A}_{GP} = 2\vec{\Omega}_{GP} \times \vec{V} \quad , \quad \vec{\Omega}_{GP} = -\frac{3}{2c^2}\frac{G\vec{M}_E}{|\vec{X}|^3}(\vec{V} \times \vec{X})$$




LAGEOS 1

$$\begin{bmatrix} +0.245 \\ +2.141 \\ +0.928 \end{bmatrix}_{XYZ_0} \cdot 10^{11}\ \mathrm{m/s^2}$$

$$\vec{A}_{LTP} = 2\vec{\Omega}_{LTP} \times \vec{V} \quad , \quad \vec{\Omega}_{LTP} = -\frac{G}{c^2 |X|^3}\left[ \vec{S}_E - \frac{3(\vec{S}_E \cdot \vec{X})\vec{X}}{|X|^2} \right]$$




LAGEOS 1

$$\begin{bmatrix} +0.13 \\ +3483 \\ +40.10 \end{bmatrix}_{XYZ_0} \cdot 10^{12}\ \mathrm{m/s^2}$$

# **Geodesy:** a modern view…

- **Classical approach:** "Newton" + relativistic corrections for precise satellite dynamics and time measurements.

  - *Advantages*: - Well-proven method.
    - Might be sufficient for current applications.

  - *Drawbacks*: - To be adapted to the adopted space-time transformations and to the level of precision of data

- ***Alternative* and *pioneering* effort:** develop a satellite motion integrator in a pure relativistic framework.

  - *Advantages*: - To easily take into account all relativistic effects with "metric" adapted to the precision of measurements and adopted conventions.
    - Same geodesic equation for photons (light signals) massive particles (satellites without non-grav forces)
    - Relativistically consistent approach

## hod: GINS provides template orbits to validate the RM

ns with 1) Schwarzschild metric => validate Schwarzschild correction

2) (Schwarzschild + GRIM4-S4) metric => validate harmonic contribu

3) Kerr metric => validate Lens-Thirring correction

4) GCRS metric with(out) Sun, Moon, Planets => validate geodetic prec

(other bodies contr

(…)

## b) RMI goes beyond GINS capabilities:

- (will) includes

1) IAU 2000 standard GCRS metric

2) IAU 2000 time transformation prescriptions

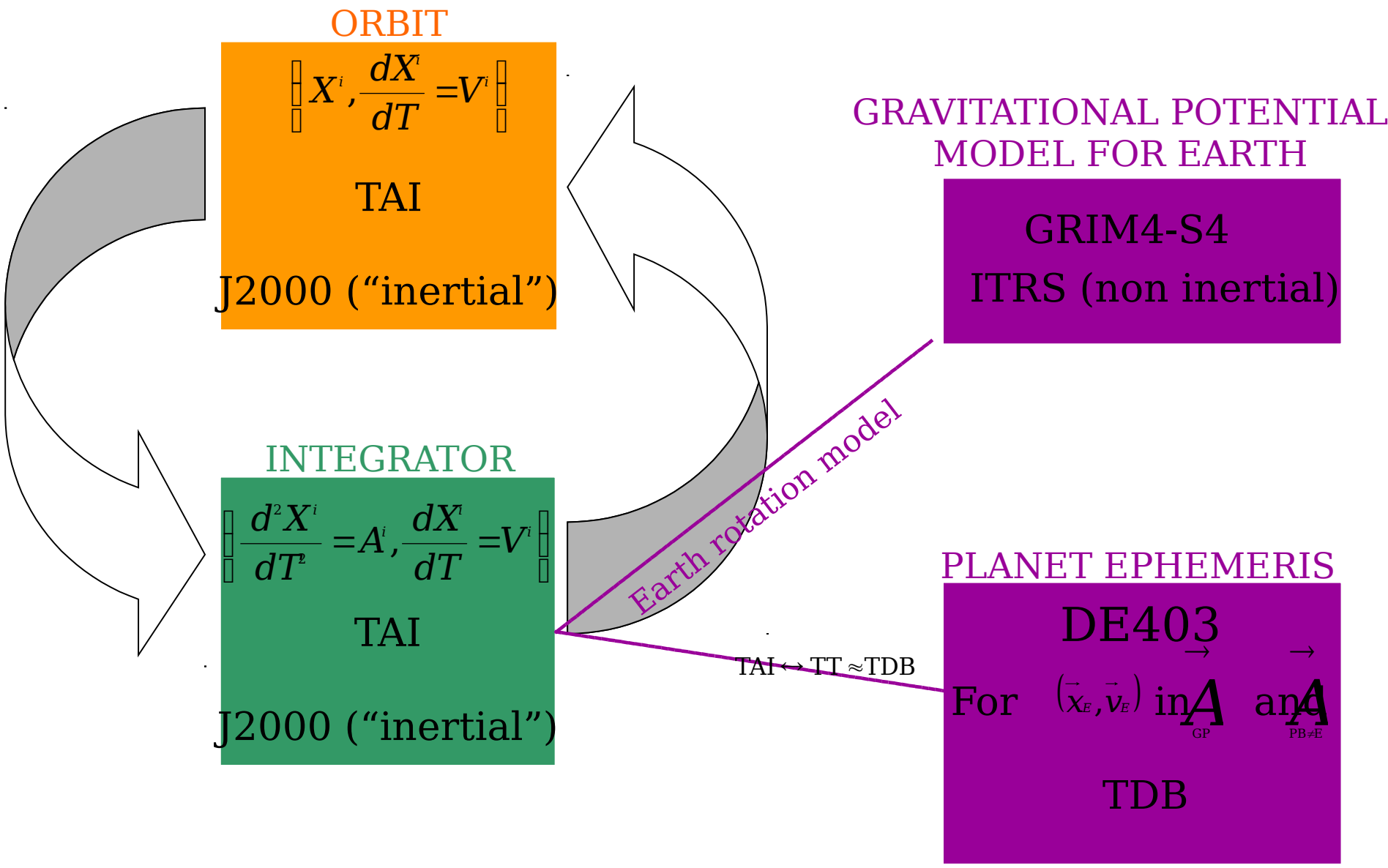
3) IAU 2000/IERS 2003 new standards on Earth rotation

4) post-newtonian parameters in metric and time transformations

- separate modules allow easy update for metric, Earth potential model (EGM96)… prescriptions

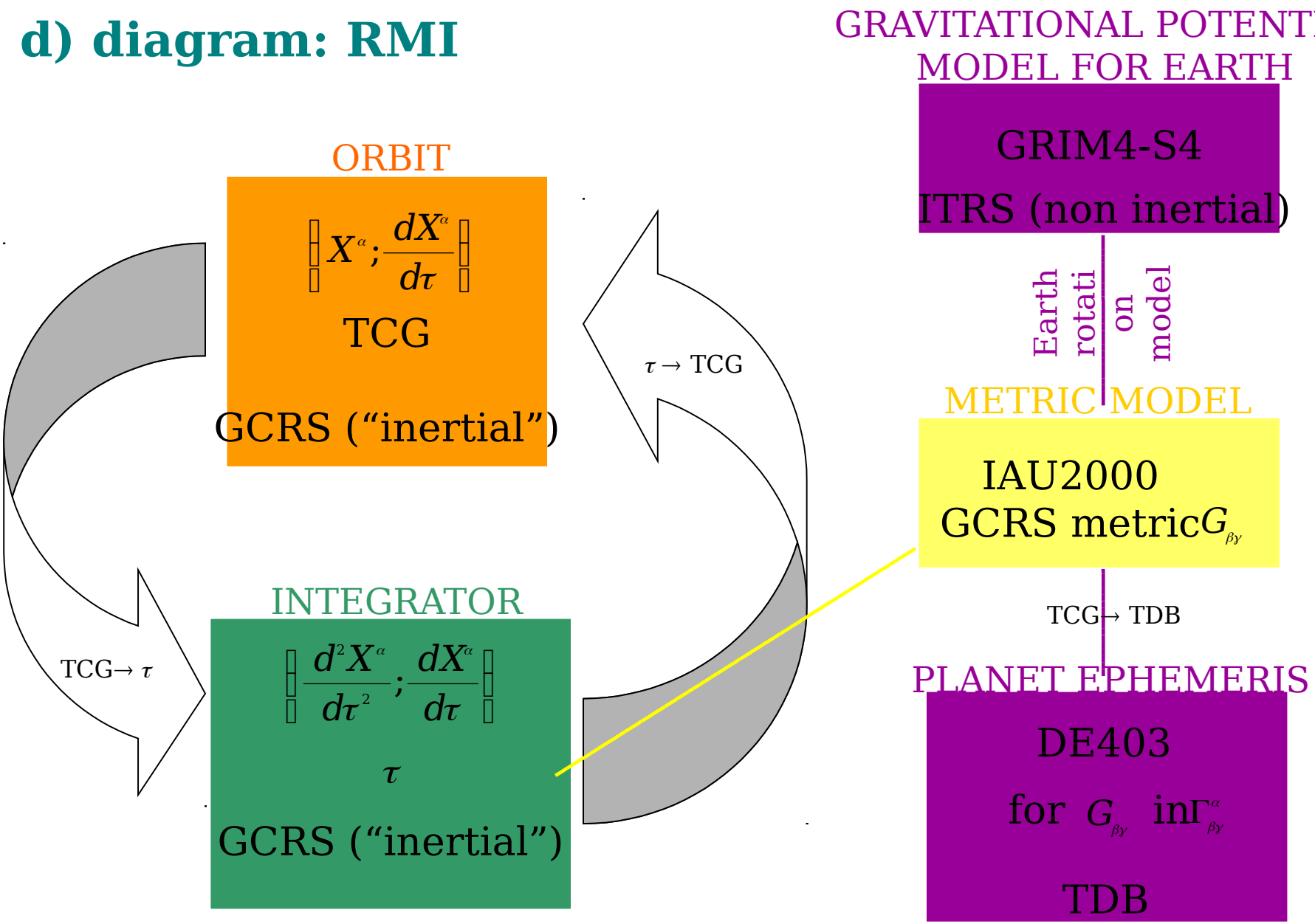
- contains all relativistic effects, different couplings at corresponding metric order

# c) diagram: GINS




with *i=1,2,3* spatial indices

# d) diagram: RMI




with $\alpha$=*0,1,2,3* space-time indices

# **Geodesy:** principle of accelerometers…

ellite Center of Mass at $X^\rho$ (generalized relativistic eq.)

-mass, shielded from non-gravitational forces, at $X^\rho + \delta X^\rho$

odesic eq.)

iff→ence between the two equations at first order in $\delta X^\rho$ :

$$-K_\beta\left[G^{\alpha\beta}-\frac{1}{c^2}\frac{dX^\alpha}{d\tau}\frac{dX^\beta}{d\tau}\right]=\frac{d^2\delta X^\alpha}{d\tau^2}+\frac{\partial\Gamma^\alpha{}_{\beta\gamma}}{\partial X^\lambda}\delta X^\lambda\frac{dX^\beta}{d\tau}\frac{dX^\gamma}{d\tau}+2\Gamma^\alpha{}_{\beta\gamma}\frac{dX^\beta}{d\tau}\frac{d\delta X^\gamma}{d\tau}$$

with $G^{\alpha\beta}$, $\Gamma^\alpha{}_{\beta\gamma}$ evaluated at $X^\rho$

$K_\nu$ for the CM of satellit

lassical limit → $-K_i=\frac{d^2\delta X^i}{dT^2}+\sum_j\frac{\partial^2 W}{\partial X^i\partial X^j}\delta X^j$

# **Geodesy:** bibliography

## Relativistic time transformations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## RMI

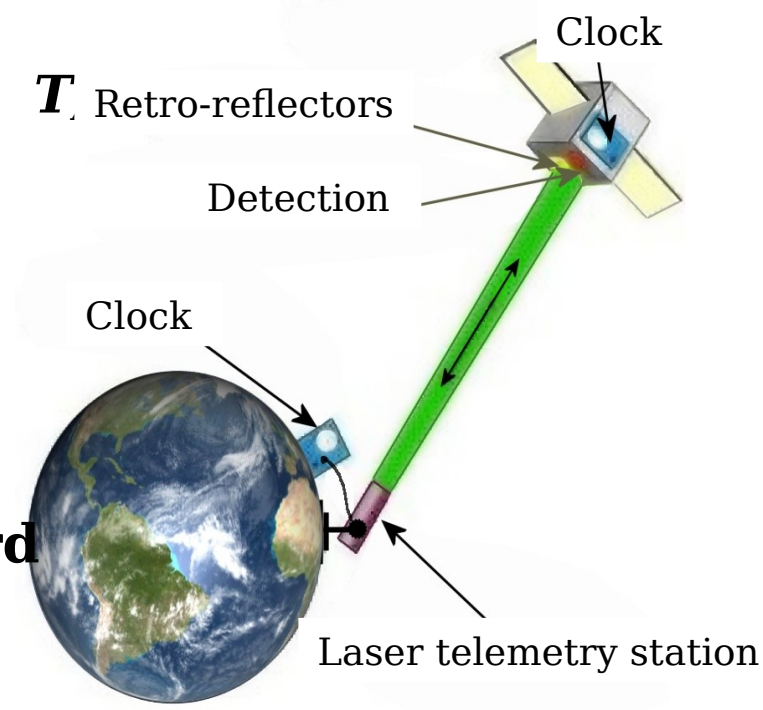

[GRGS 2001] *Descriptif modèle de forces: logiciel GINS*
[Moisson 2000] *(thèse).* Observatoire de Paris
[McCarthy Petit 2003] *IERS conventions 2003*

*http://maia.usno.navy.mil/conv2000.html*.

# T2L2 (optical telemetry with 2 laser links)

## Principle of *ground-space* time transfer:

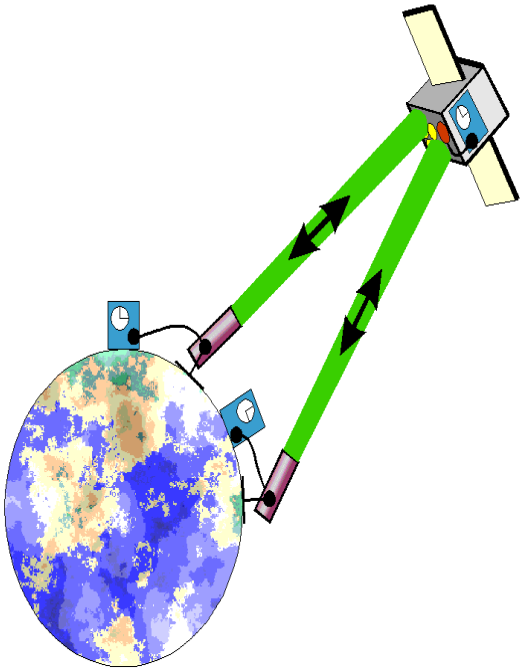
- **Date laser pulses:**
  - Departure from ground station: $T_A$
  - Arrival aboard: $T_{sat} = T_B$
  - Echo return on ground: $T_C$




- **Follow evolution of time aboard *wrt* ground time:**
  - Rebuild triplets ($T_A$, $T_{sat}$, $T_C$)
  - Compute ground-satellite delay:

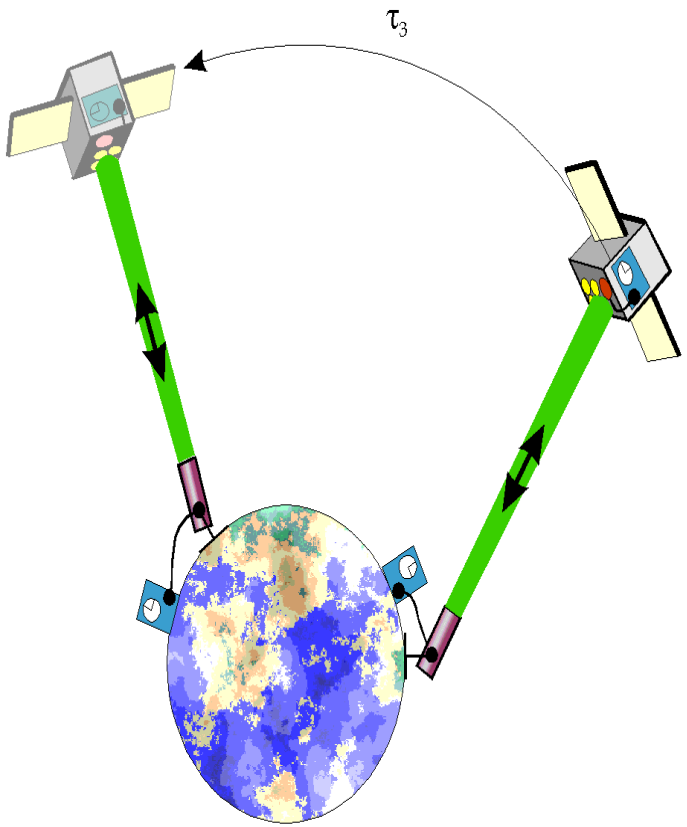
$$\left[ T_A + \frac{T_C - T_A}{2} + \frac{dT_{A\text{-}sat\text{-}C\,relativist}}{2} + \frac{dT_{A\text{-}sat\text{-}C\,atmosph}}{2} + \frac{dT_{A\text{-}sat\text{-}C\,calib}}{2} \right] - T_{sat}$$

# Principle of *ground-ground* time transfer:



**Common view**

On-board oscillator noise $\sigma_x(0.1\ s)$



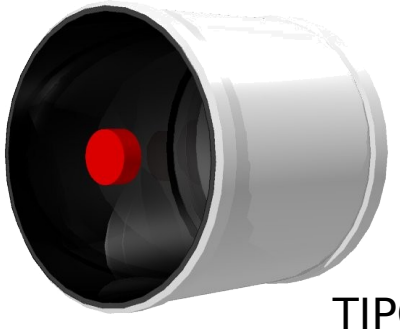

**Non-Common view**

On-board oscillator noise $\sigma_x(\tau_3)$

# TIPO (Télémétrie Interplanétaire Optique)

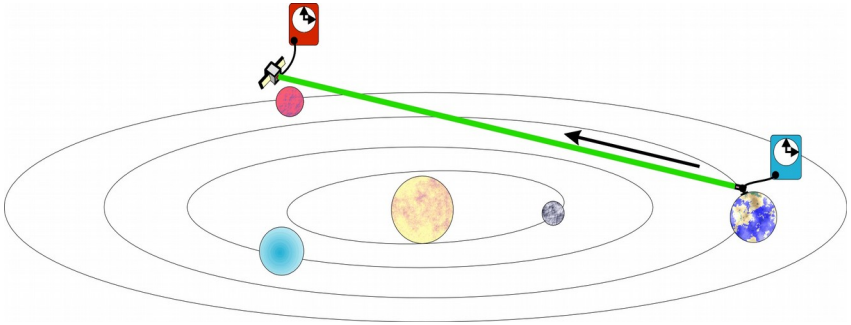
## Scientific objectives of TIPO:

- Mesure PPN parameter $\gamma$ (Shapiro effect)
- Planet Telemetry
- Asteroid masses
- Pioneer effect
- …



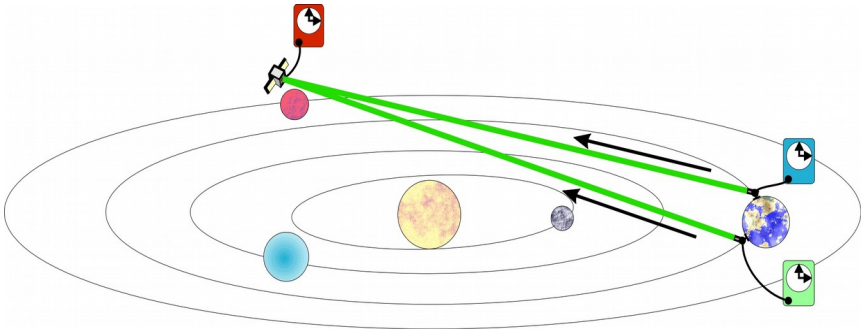
TIPO Telescope

## Method:



**Radial distance measurement**

$\sigma_x = 2.6 \cdot 10^{-13} \cdot \tau^{1/2}$ : centimetric over 1 day

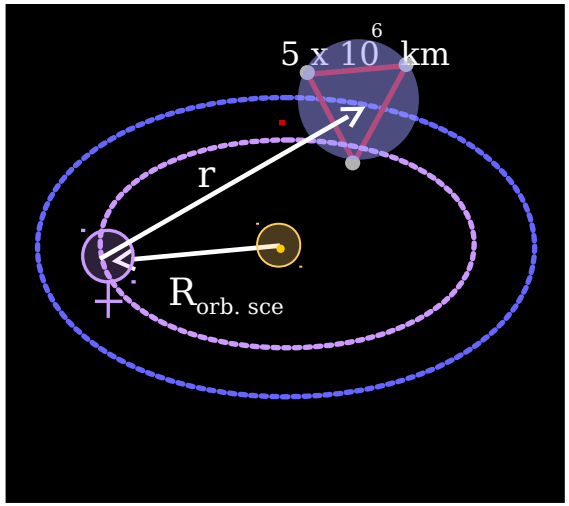


**Angular distance measurement**

$\delta\theta = 2.10^{-9}$ rd

**bital motion of sces during photon flight time:**

$$\delta\left[\frac{2GM}{rc^2}\right] \sim \frac{2GM}{rc^2}\,\frac{2\pi\,\mathrm{R}_{\mathrm{orbsce}}}{\mathrm{r}}\,\frac{\delta t_{vol}}{T_{\mathrm{sce}}}$$




with $\dfrac{\mathrm{R}_{\mathrm{orb.sce}}}{\mathrm{r}} \sim 1$ for planets, << 1 for Sun

# **T2L2,** rotation around the Earth:

$$H_{\alpha\beta} = {}^{(1)}H_{\alpha\beta} + {}^{(3/2)}H_{\alpha\beta} + {}^{(2)}H_{\alpha\beta} + \dots$$

$\sim H^2$ $10^{-18}$

**Earth rotation:** $\frac{4\Omega GM}{c^3}\frac{R^2}{r^2}$ $\sim 10^{-15}$

**orbital motion of sces :** $\frac{4GMV_{sce}}{c^3 r}(\times 1 ou \frac{R}{r})$

Sun $\sim 10^{-15}$

Moon $\sim 10^{-18}$

Jupiter $\sim 10^{-19}$

$$= {}^{(1)}H_{\alpha\beta}^{(Earth)} + {}^{(1)}H_{\alpha\beta}^{(othebodies)}$$

$$\sim H\frac{m}{M_T}\frac{R}{r}\times\frac{R}{r}$$

Sun $\sim 10^{-11}$

Moon $\sim 10^{-13}$

Jupiter $\sim 10^{-15}$

$$= {}^{(1)}H_{\alpha\beta}^{(sphEarth)} + {}^{(1)}H_{\alpha\beta}^{(nonsphEarth)}$$

$$\frac{2GM}{rc^2}.(J_2) \quad \sim 10^{-12}$$

$$\frac{2GM}{rc^2}.(J_3,\dots) \quad \sim 10^{-15}$$

$$\sim \frac{2GM}{rc^2} \quad \sim 10^{-9}$$

**vol photon:** **0.1 s** $\sim 10^{-10}$ s

# Collaborations in LISA FRANCE

ISA France:

- **APC, Paris 7**
- **ARTEMIS, OCA**
- CNES
- IAP Paris
- LAPP Annecy
- LUTH Observatoire de Paris-Meudon
- ONERA
- Service d'Astrophysique CEA

## ARTEMIS, OCA:

**Chauvineau**: gravitation relativiste

**Pireaux**: gravitation relativiste, théories alternatives

**Régimbau**: modélisation d'ondes gravitationnelles - fond stochastique

**. Vinet**: Time-Delay Interferometry